We can easily determine the solution of Eq. (4). We get for the resolvent:

$$R(s) = [C_1(\pi s)^{1/2} + 1][s + C_1(\pi s)^{1/2} + 1]$$

Applying Jordan's Lemma[3] one can show that the integral in Eq. (6) is equal to the sum of the residuals (Res) of the integrand. Thus we get

$$u_p(\tau^*) = \int_{\tau_0}^{\tau^*} \Big\{ n(t) - \int_{\tau_0}^{t} \sum_{i=1}^{2} \mathrm{Res}[R(s_i)\exp(s_i x)] \Big|_{x=t-\tau'} n(\tau')\mathrm{d}\tau' \Big\} \mathrm{d}t \qquad (7)$$

By the method of decomposition in partial fractions, one can show that the denominator polynomial is of second degree. It has the form

$$f(s) = s^2 + (2 - C_1^2\pi)s + 1 \qquad (8)$$

It follows from an order-of-magnitude estimation of the coefficients that the two roots are conjugated complex. Their real parts lie in the left half-space of the complex plane:

$$s_{1,2} = (C_1^2\pi/2) - 1 \pm i[1 - (2 - C_1^2\pi)^2/4]^{1/2} \qquad (8a)$$

We note that, for large particle-density ratio ($\sigma \gg 1$) and negligible lift forces, $C_1 \cong 3/2.\ (2/\pi\sigma)^{1/2}$, while for $\sigma \approx 1$, $C_1 = (2/\pi)^{1/2}$. In the latter case, we get asymptotic stability:

$$s_{1,2} = \pm i \qquad (8b)$$

One can recognize that, generally, the Basset term in the BBO equation gives rise to damped oscillations of the particle. However, with increasing particle rotation velocity and spatial gradients of the mean flow, velocity C_1 rises and the Basset term leads to unstable modes. This happens also for absent lift force when $2A_1$ is less than $81/(\sigma + 0{,}5)^2$. Applying the Laplace-transformation directly to Eq. (3), we get

$$U_p(s) = \frac{U_F(s)[Bs + C_1(\pi s)^{1/2} + 1] + D_1 s}{s[s + C_1(\pi s)^{1/2} + 1]} \qquad (9)$$

with

$$\mathcal{L}(a_p) = A_p(s),\ \mathcal{L}(r) = R(s),\ \mathcal{L}(n) = N(s),\ \mathcal{L}(K) = L(s)$$

Conclusions

From Eq. (9) the amplitude and phase ratio can be determined immediately by complex calculations setting $s = j\omega$. The results are presented in Hinze,[4] using a Fourier integral representation of $u_p(t)$ and $u_F(t)$. It can easily be shown that the determinant of the antimetric transfer matrix of the amplitudes of particle and fluid motion is equal to the ratio of the Lagrangian energy-spectrum functions. For short diffusion times, the main contribution to the diffusion coefficient comes from the high-frequency components of motion. The smaller the marker-particles, the shorter the particle relaxation time. This indicates to what extent the particle follows the high frequency fluctuations of the fluid.

Corrections Concerning Ref. 1

The equations labeled as in Ref. 1 read correctly as follows:

Eq. (2): $\dfrac{\mathrm{d}U_p}{\mathrm{d}\tau} + AU_p = B\dfrac{\mathrm{d}U_F}{\mathrm{d}\tau} + AU_F + D$

In Eqs. (3), (6), and (7): the dynamic viscosity μ_F in the coefficient $D = d^2g/\mu_F$ should be substituted by the kinematic viscosity ν_F.

Eq. (15): $f \simeq n_1 f_1 + n_2 f_2 + \ldots + n_N f_N$

Eq. (17): $U_{pj}(\tau^*) = \tilde{A}_j[(1/\tilde{\omega}_j^* - B)\sqrt{2}]\cos(\tilde{\omega}_j^*\tau^* + \Theta_j') + D_1$

Eq. (18): $\dfrac{U_{pj}}{U_{Fj}} = \dfrac{1/\omega_j^* - B}{(\sqrt{2})^{-1}} - \dfrac{D_1}{\tilde{A}_j}\mathrm{sgn}(1/\omega_j^* - B)$

In Eq. (20): $\mathrm{sgn}\left(\dfrac{1}{\omega_j^* - B}\right)$ must be subtituted by

$\mathrm{sgn}\left(\dfrac{1}{\omega_j^*} - B\right)$

In Eq. (21): the factor $\dfrac{\sigma d g^2}{18\mu_F}$ by $\dfrac{\sigma d^2 g}{18\mu_F}$

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Reply by Author to F. Ebert and S. U. Schöffel

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THE author wishes to express his appreciation to Ebert and Schöffel for their observations of typographical errors in equations described in a previous article.[1] These corrections certainly enhance the usefulness of the results. In addition, the author wishes to comment on the assumptions used in the development.

Consider the Basset-Boussinesq-Oseen equation depicted in Eq. (1) in the author's article. The form of the equation suggests the consideration of the relative magnitudes of the different terms and a resultant simplification. For the case when the density of the flow tracing particle is substantially greater than the density of the fluid, as is commonly the case in laser velocimetry (LV) applications, it is possible to neglect the effect of the added mass and to set the added mass coefficient equal to zero.[2] Also, again for the special case of interest to

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the experimentalist using an LV, the Basset integral term may be neglected since C is small compared to the other coefficients in Eq. (1). Lastly, the lift force is incorporated into the coefficient D, only for generality.

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Comment on "Buckling of Composite Plates with a Free Edge in Edgewise Bending and Compression"

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TABLE 2 of Ref. 1 presented buckling loads for a laminated rectangular plate with two loaded edges simply supported, one side free and the opposite side either simply supported or clamped, for linearly varying edge loads. Since the analysis of Ref. 1 is based on classical plate theory, it is of interest to compare those results with buckling loads that include the effect of transverse shear deformations.

Such buckling loads have been presented for plates with uniformly loaded edges.[2] These were calculated using the shell-of-revolution program FASOR[3] by considering the plate as part of a cylinder of very large radius. The circumferential direction of the cylindrical model corresponds to the longitudinal (loaded) direction of the plate, and the axial length of the cylinder equals the plate width b. The plate buckling load is obtained by minimizing the model buckling load with respect to the circumferential wave number N subject to the condition $N = n\pi R/a$, where n is the number of longitudinal half-waves of the plate buckle, R is the radius of the cylindrical model, and a is the length of the plate. This procedure is also applicable to plates with linearly varying edge loading.

The dimensions of the laminated plate analyzed in Ref. 1 are $a = 254$ mm (10 in.) and $b = 50.8$ mm (2 in.). It is made of 0.127-mm (0.005-in.)-thick tape with the following in-surface elastic properties:[4] $E_1/E_2 = 10.05$, $G_{12}/E_2 = 0.349$, $\mu_1 = 0.34$, and $E_2 = 13.03$ GPa (1.89×10^6 psi), where 1 and 2 signify directions parallel and transverse, respectively, to the fibers and μ_1 is the major Poisson's ratio. The laminate definition is $[\pm 45_3/0_3]_s$ (erroneously reported as $[\pm 45_3/0_3/90_3]_s$ in Ref. 1), thus giving a laminate thickness $h = 2.286$ mm (0.090 in.) Neglecting the small anisotropic effect, this laminate has the bending stiffness matrix given by Eq. (25) of Ref. 1.

Since the values of the transverse shear moduli G_{13} and G_{23} are unavailable, it is assumed that $G_{13} = G_{23} = G_{12}$. (For typical unidirectional laminae, $G_{23} < G_{12}$.) Again neglecting the small anisotropic effect, FASOR gives the transverse shear stiffness matrix[5] $[K] = 8.951 \times 10^6$ N/m (5.112×10^4 lb/in.) $[I]$, where $[I]$ is the 2×2 unit matrix.

Table 1 compares the classical plate theory results of Ref. 1 with the transverse shear deformation theory results calculated by FASOR. It is noted that the buckling load knockdowns due to transverse shear deformation for this plate are smaller than those reported for uniformly compressed plates in Ref. 2. Aside from differences in laminar properties, lay-up, and edge conditions, this is not surprising since the thickness-to-width ratio for this plate is $h/b = 0.045$, which is less than one-half that of the thinnest plate studied in Ref. 2.

Table 1 Buckling stress resultants $N_x b^2/E_2 h^3$ at free edge ($y = b$)

	Condition at supported edge ($y = 0$)					
	Simply supported			Clamped		
$N_x(0)/N_x(b)$	CPT[a]	FASOR[b]	Diff., %	CPT[a]	FASOR[b]	Diff., %
100	0.101	0.097	4.0	0.219	0.202	8.6
2	2.106	2.018	4.4	4.221	3.954	6.8
1	2.637	2.525	4.4	5.133	4.792	7.1
0.5	3.016	2.886	4.5	5.745	5.357	7.2
0.01	3.507	3.356	4.5	6.491	6.042	7.4
−0.5	4.221	4.036	4.6	7.487	6.952	7.7
−1	5.263	5.027	4.7	8.762	8.121	7.9
−2	10.033	9.579	4.7	12.777	11.744	8.8

[a]Classical plate theory (Ref. 1). [b]Transverse shear deformation theory.

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Comment on "Stiffness Matrix Adjustment Using Mode Data"

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THE above paper[1] is a welcome addition to the literatures of structural system identification. The author presents a stiffness matrix adjustment (KMA) procedure that shows some promising results. Nevertheless, we believe that the following comments are appropriate for further developments in the field, especially in relation to real large structures.

Let p be the number of constraints (equations) and q the number of unknowns to be identified. Depending on relative values of p and q, there are essentially two kinds of system identification.[2,3] For an overdetermined system, $p > q$, a least-square solution is sought that minimizes errors of the solution. The procedures given in Refs. 4 and 5 fall into this category. On the other hand, if $p < q$ (underdetermined system), there are infinitely many sets of solutions satisfying given con-

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